Prudential fiscal stimulus*

Alfred Duncan[†] Charles Nolan[‡]
University of Kent University of Glasgow

This version: October 2025

When government fiscal interventions are predictable, private incentives are distorted. If firms anticipate fiscal stimulus in a crisis, then they might take on excessive risk today. How can fiscal interventions be designed to mitigate such moral hazard problems? And would such interventions be time consistent? We show that fiscal stimulus programmes can be designed to induce precautionary behaviour ex ante from firms, and we label these programmes *prudential fiscal stimulus*. We demonstrate the theory with a wage subsidy stimulus policy. We show that countercyclical wage subsidies can be welfare improving even in the absence of aggregate demand or labour market externalities. Prudential fiscal stimulus is time inconsistent, but the presence of aggregate demand externalities can bring discretionary policies closer to optimal policies.

JEL Classification: E32, D52.
Keywords: Macroeconomics, Fiscal Stimulus, Incomplete Markets.

*All errors are our own. This work was supported by UKRI Grant Ref: ES/V015559/1. We are very grateful for helpful feedback from and discussions with seminar and conference attendees at the University of Auckland, the University of Kent, the University of Wellington, the Bank of Lithuania, the Reserve Bank of New Zealand, the 2022 Money Macro Finance Society Annual Conference, the 2022 Scottish Economic Society Annual Conference and especially Campbell Leith.

[†]Corresponding author. Email: a.j.m.duncan@kent.ac.uk
[‡]Email: charles.nolan@glasgow.ac.uk

Introduction

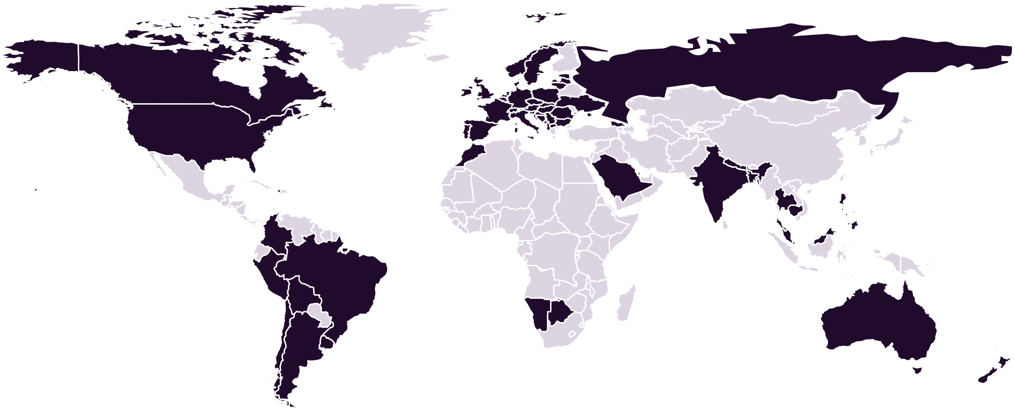
During the Panic of 1857, the Bank of England instructed clerks in their Newcastle branch to lend "to concerns employing large numbers of hands, to the extent required for the payment of wages," even upon securities "not of a strictly Banking character" (Bank of England, 1858, Entry: 26 November 1857). The extension of irregular credit to support wage bill payments of local businesses followed the decision to refuse credit to the troubled Northumberland and Durham Bank, despite the assessment that the bank did hold sufficient pledgeable high quality but illiquid bills, and that their unmanaged failure would generate significant wider costs. In their evidence to the Select Committee on the Bank Acts, the Bank justified their decision to refuse credit to Northumberland and Durham, even for a managed liquidation, by appealing to moral hazard concerns. Northumberland and Durham had borrowed from the Bank in 1847; they had taken imprudent risks overexposing themselves to a single creditor; they attempted to put pressure on the Bank by appealing to the wider economic damage that might result from their failure (UK House of Commons, 1858).

The Bank sought to have it both ways: to support the economy in crisis while leaning against moral hazard. In this paper, we show that it is possible to have it both ways. Stimulus in crises doesn't need to exacerbate moral hazard in expansions; it can even reduce it. Wage subsidies in particular are a form of what we call *prudential fiscal stimulus*: ex post stimulus that encourages ex ante prudence.

Prudential fiscal stimulus policies increase welfare even in the absence of aggregate demand externalities. In our benchmark model, stimulus policies are prudential when they complement precautionary behaviour by firms (including banks) in good times. Our primary example is a simple rule for the design of a wage subsidy. That rule conditions the wage subsidy on real GDP; when GDP is low, the wage subsidy is high, and vice versa. Labour is a complement to firms' inside wealth (entrepreneurs' own equity capital), and the countercyclical wage subsidy increases the marginal value of firms' inside wealth in downturns. That encourages firms to take precautionary actions in good times, ensuring that they can take advantage of the wage subsidy in downturns. In a banking extension of our model, the wage subsidy similarly increases the marginal value of bank equity in downturns. In short, the prudential stimulus nullifies the usual concerns with government intervention; instead of blunting beneficial precaution, the wage subsidy boosts productive capacity in downturns by encouraging firms (banks) in good times to accumulate more inside wealth than they otherwise would.

Wage subsidies are not the only prudential fiscal stimulus policy in our model. Our focus on wage subsidies is motivated in part by the widespread use of related schemes during the Covid-19 pandemic. Based on ILO (2020) and IMF (2021) reports, we calculate that 72 countries (the regions shaded in Figure 1) have used either existing or new wage subsidy and job retention subsidy schemes as part of their economic response to the Covid-19 pandemic.[1]

Figure 1: Countries with new or existing wage subsidy schemes during the Covid-19 pandemic



Sources: The International Labour Organization Organization (2020), the International Monetary Fund (2021), authors' calculations. Wage subsidy programmes are defined as any programme that provides wage support to employers to retain workers in employment.

[1]By May 2020, wage subsidy and job retention schemes supported about 50 million jobs across the OECD, about ten times as many as during the global financial crisis. Schemes varied substantially across countries in terms of duration, generosity and design. The wage subsidy we model is inevitably a somewhat stylized version of those actually implemented. See https://www.oecd.org/coronavirus/policy-responses/ supporting-jobs-and-companies-a-bridge-to-the-recovery-phase-08962553/ for some details on design. Linden et al. (2021) look at the design and impact of different wage subsidy schemes on various metrics: The Structure and Incentives of a COVID related Emergency Wage Subsidy Jules Linden, Cathal O'Donoghue, Denisa M. Sologon. See also our discussion later.

Model overview and intuition

Our model examines how financial frictions affect entrepreneurial risk-bearing and the role of fiscal policy in mitigating these risks. These frictions force entrepreneurs to bear residual project risk but do not preclude trade in aggregate risk with households.

During periods of financial stress, entrepreneurs' inside wealth earns high expected returns with increased uncertainty. This increased risk results in entrepreneurs discounting the associated high returns, damping the market's signal to recapitalise. Ex ante, entrepreneurs accept a larger share of aggregate risk in competitive allocations than in constrained efficient allocations. This distortion motivates a role for macroprudential policy to internalize the social costs of fluctuations in firm leverage.

In the absence of optimal macroprudential policies, ex post interventions that increase the marginal value of entrepreneurial wealth during downturns can restore incentives for ex ante entrepreneurial prudence. A countercyclical wage subsidy serves this purpose.

Our analysis focuses on the incentive effects of fiscal policy, ensuring that policy interventions do not merely replicate markets that private agents would wish to trade in but cannot due to ad-hoc market closures. By addressing these financial frictions and their implications, we demonstrate how prudential fiscal stimulus can enhance overall economic efficiency.

The main result of the paper is Proposition 1, which derives a closed-form expression for the optimal state contingent labour subsidy (or tax) in a non-linear DSGE model supporting a range of underlying financial frictions. Optimal wage subsidies increase on impact from shocks that tighten financial frictions, but the size of the optimal wage subsidy is moderated by historical shocks: wage subsidies are most effective if introduced quickly in response to shocks.

The specific financial friction that we introduce for quantitative analysis is the imperfect state verification model based on Duncan and Nolan (2019). This model provides a tractable foundation for the optimality of standard debt contracts without the need for additional market incompleteness that doesn't necessarily follow from information asymmetries. We estimate the nonlinear model and use the estimated model to quantify an optimal wage subsidy simple rule responding only to output deviations from the steady state. We find that a wage subsidy simple rule with an elasticity on output of 1.8 maximises expected welfare gains, and reduces the expected welfare costs of business cycles by 20 percent relative to the laissez-faire counterfactual. Expected welfare gains are larger for parameterisations where more of the amplitude and persistence of fluctuations is due to financial frictions rather than the underlying shocks.

Our benchmark model only includes one friction, and our policy instrument acts on the labour market, a market that is subject to a number of distortions in practice. We undertake two extensions to verify that our results hold in the presence of further labour market distortions. In Section 4.2 we impose a constant labour tax. This increases the optimal size of wage subsidies and the resulting welfare gains. In Section 5 we introduce New Keynesian price rigidities. This generate a time varying labour market wedge of inefficiency. We demonstrate that wage subsidies are part of the efficient policy response to shocks.

In our flexible price model, prudential fiscal stimulus is not time consistent. When the downturn arrives, policymakers have no incentive to introduce a distortionary stimulus policy. Under sticky prices, aggregate demand externalities in downturns can restore the time consistency of stimulus policies, even if the welfare gains from the stimulus are still largely driven by their prudential effects. It is well known that acting under discretion, policymakers have an incentive to accommodate supply shocks, but that the rational expectation of this policy stimulus may worsen outcomes (Barro and Gordon, 1983; Kydland and Prescott, 1977). In our New Keynesian model, the rational expectation of prudential fiscal stimulus increases the efficiency impact of the stimulus, through the resulting increase in retained earnings during expansions.

1 The model

The model consists of a population of firm owner entrepreneurs, and a population of identical worker households. Entrepreneurs have access to a special production technology that generates stochastic output from labour and capital, where the aggregate capital stock is fixed at unity. Entrepreneurs and households both have risk averse preferences with respect to consumption—although the extent of risk aversion is allowed to differ across groups in our quantitative analysis.

An individual entrepreneur's production y is subject to idiosyncratic shocks, $y^e = \theta z (k^e)^{1-\alpha}(h^e)^\alpha$ where θ is a privately observed individual-specific shock, z is a common total factor productivity shock, and k^e, h^e are the quantities of capital and labour employed by the individual entrepreneur. The shock θ takes values $\{\bar{\theta}, \underline{\theta}\}$ with mean one and $\bar{\theta} > \underline{\theta}$, where the probabilities $P(\theta)$ are fixed but the difference $\bar{\theta} - \underline{\theta}$ is time varying in response to a common knowledge risk shock. Aggregate production is given by $y = zh^\alpha$. Information asymmetries are assumed to limit the extent to which entrepreneurs can defray risk to outside investors. Beyond information asymmetries, there are no other ad hoc restrictions on financial markets within periods—entrepreneurs could always raise additional funds from noncontingent loans, but leverage is limited by entrepreneurs' capacity for risk bearing in equilibrium. We first set out the entrepreneurs' and households' approach to aggregate risk sharing. Then we turn to the implications of the contract required to encourage households to invest in firms given the asymmetric information, costly and error-prone state verification.

1.1 Entrepreneurs

We decompose the entrepreneurs' problem into two parts. At the end of each period, after realising output and repaying any within period loans and labour contracts, entrepreneurs choose their consumption and trade in aggregate risk securities. At the beginning of the following period, aggregate risks are realised, entrepreneurs' aggregate risk securities are paid, and entrepreneurs hire labour and borrow to fund capital acquisition for production.

At the end of the period, an entrepreneur faces the following problem:

Programme 1 *At the end of the period, subsequent to the realisation of individual-specific states (θ, ϑ), an individual entrepreneur solves the following problem:*

$$v^e(q^e) = \max_{x^e, c^e, q^{e\prime}} \mathbb{E}\left\{\log c^e + \beta^e v^e(q^{e\prime})\right\}$$

subject to the resource constraint,

$$q^{e\prime} = R(\theta, \vartheta)q^e - c^e - \int_{s\in S} p(s)x^e(s)ds + x^e(s'),$$

where $R(\theta, \vartheta)$ describes the return to an entrepreneur's inside wealth q^e, conditional upon individual-specific states θ and ϑ. The mapping R is determined by the optimal within-period financial contract, solved in Programme 2.

For clarity on timing, q^e denotes wealth after all aggregate and idiosyncratic uncertainty in period t has been resolved, while $q^{e\prime}$ denotes wealth after period $t+1$ aggregate and idiosyncratic uncertainty is resolved. The expectation $\mathbb{E}$ is formed in period t over the realisations of aggregate and individual specific states in periods $t+1, t+2, \ldots$. Trade in aggregate risk markets is captured by the quantities $x^e(s)$, denoting the amount purchased of an asset with payoff 1 conditional upon the future state of the world being realised as state s. The current period price of this security is denoted $p(s)$. As indicated earlier, trade in securities indexed by the aggregate state are not hampered by any problem of asymmetric information; unlike idiosyncratic states, aggregate states are costlessly observed and verified by all agents. These markets are active.

Our ability to separate an entrepreneur's problem in this way depends on our use of information asymmetries only as the source of market incompleteness in our within period financial market. Collateral constraints, or similar, that restrict borrowing within period would also restrict agents' choices of aggregate risk securities x^e.

Programme 2 *At the beginning of the period, an individual entrepreneur solves the following problem:*

$$R(\theta, \vartheta) = \arg \max_{b^e, h^e, k^e} \mathbb{E}_\Theta \log R(\theta, \vartheta)$$

subject to the capital constraint,

$$Qk^e = q^e + b^e,$$

the truth-telling constraint,

$$R(\underline{\theta}, \overline{\vartheta})q^e + (\overline{\theta} - \underline{\theta})z(k^e)^{1-\alpha}(h^e)^\alpha \leq R(\overline{\theta}, \emptyset)q^e, \tag{1}$$

and the lenders' breakeven constraint,

$$\sum_{(\theta,\vartheta)} P(\theta, \vartheta)\left(\theta z(k^e)^{1-\alpha}(h^e)^\alpha - (R(\theta, \vartheta) - 1)q^e\right) \geq wh^e + rb^e + P(\underline{\theta})\kappa k^e. \tag{2}$$

where the expectation $\mathbb{E}_\Theta$ is formed before the current period realisation of individual specific shocks θ, ϑ.

The first constraint defines the value of capital contributing to an individual entrepreneur's project as the sum of their initial wealth and funds borrowed by the entrepreneur from households. Equation 1 is the incentive compatibility constraint. $R(\underline{\theta}, \overline{\vartheta})q^e$ denotes the gross income received by an entrepreneur whose productivity shock is wrongly revealed to be the high type. $R(\overline{\theta}, \emptyset)q^e$ denotes the gross income received by an entrepreneur who receives the high type productivity shock. Equation 2 is the households' participation constraint. Repayments to external financiers are contingent on the idiosyncratic shock θ as well as any audit signal obtained by the household (or bank), $\vartheta \in \{\underline{\vartheta}, \overline{\vartheta}\}$. Audit signals are distributed as follows: $P(\overline{\vartheta}|\overline{\theta}) = 1, P(\overline{\vartheta}|\underline{\theta}) \in (0, 1)$, so that high types are always correctly identified by the audit, but with some probability low types are incorrectly tagged as high types. Thus to access outside investment, entrepreneurs need to commit to overturned low reports paying out more than high reports. This induces truth-telling and is ensured by the incentive compatibility constraint. To ensure the investment of households, expected loan repayments must exceed the sum of the households' opportunity cost of funds, rb^e plus expected audit costs, $P(\underline{\theta})\kappa K$, where κK is the cost of each audit for some constant κ. It turns out that that when audit costs are sufficiently low, defaultable debt contracts with deterministic audit strategies are optimal (Duncan and Nolan, 2019).

In the extension of our model, focusing on banks/financial intermediaries, we use a different microfoundation for within period loans. The important features of within period loans for our analysis are that (1) idiosyncratic risk cannot be fully passed on to outside investors, and (2) borrowing decisions are interior, with entrepreneurs always able to borrow an additional dollar on a non-contingent basis. Microfoundations based on information asymmetries alone are well suited to generate these two features.

The next few sections focus on three key features of the competitive equilibrium. First, the entrepreneurs' aggregate risk portfolio choices are combined with those of households to examine economy-wide, competitive allocation of aggregate risk. Despite these markets being open it is shown that the financial accelerator is not closed down, even for TFP shocks. Second, some details are provided on the financial contract to show intuitively why, entrepreneurs having taken on excessive risk from a societal perspective, a wage subsidy can be an effective countervailing instrument. Finally, it is shown that the financial friction results in a wedge in factor prices–less labour and capital is hired than otherwise would be the case. Moreover, there is a strongly positive relationship between equilibrium leverage and the factor price wedge.

1.2 Households

The problem for the representative household is

$$v(q) = \max_{x,c,h,q'} \mathbb{E}\left\{u(c,h) + \beta v(q')\right\}$$

subject to

$$q' = (1+r)q + wh - c - \int_{s\in S} p(s)x(s)ds + x(s')$$

Household utility is defined over consumption and labour, h. Households supply labour and use the income gained along with accumulated wealth, q, and the payoff from their Arrow-Debreu portfolio, to save, invest and consume. Households have access to a risk-free asset that pays off r per unit invested.

1.3 Financial market clearing

Aggregate risk insurance markets allocate risk between the household and entrepreneurial sectors. The sum of Arrow securities contingent on any possible future state of the world is equal to zero.

$$x^e(s) + x(s) = 0 \qquad \forall s \in S.$$

1.4 Intertemporal risk sharing

Intuitively, the equity risk premium creates a wedge in aggregate risk sharing: High expected equity returns would normally attract capital to entrepreneurs through aggregate risk markets. However, these high returns merely compensate for idiosyncratic risk. Entrepreneurs heavily discount such risky returns during downturns, preventing aggregate risk markets from fully recapitalizing firms.

The result of the trade in aggregate risk markets is the following equilibrium risk sharing condition

$$\frac{\beta^e}{\beta}\frac{u^{e\prime}(\bar{c}^e)}{u'(c)}\frac{\mathbb{E}_\Theta R(\theta,\vartheta)}{1+r} = \frac{u^{e\prime}(\bar{c}^e_{-1})}{u'(c_{-1})}, \tag{3}$$

where $\mathbb{E}_\Theta[R(\theta,\vartheta)]/(1+r)$ denotes the gross equity risk premium within the period, and $\bar{c}^e$ is the aggregate consumption of entrepreneurs. In the absence of individual specific risk, Equation 3 would collapse to the traditional perfect risk sharing condition $\big(\beta^e u^{e\prime}(\bar{c}^e)/(\beta u'(c)) = u^{e\prime}(\bar{c}^e_{-1})/u'(c_{-1})\big)$. The equity risk premium is equal to the difference between entrepreneurs' expected consumption marginal utility and the marginal utility of expected entrepreneurial consumption. The derivation of Equation 3 is in Appendix A.

The interpretation of Equation 3 is important. It seems natural to intuit that aggregate risk markets would, in effect, recapitalize firms in downturns as Krishnamurthy (2003), Nikolov (2014) and Carlstrom et al. (2014) show in workhorse financial accelerator models of Kiyotaki and Moore (1997) and Bernanke et al. (1999). If aggregate risk markets did indeed act this way then financial amplification, and the necessity for remedial policies, essentially disappears. However, that is not the case here. When entrepreneurs are risk averse and optimal leverage is interior, the high return to inside wealth during a downturn is countered by high costs of production in the form of risk premia. That is, a low level of aggregate productivity combines with the risk of low idiosyncratic productivity leading entrepreneurs to value consumption smoothing relatively highly compared with financial stabilisation (that is, accruing inside wealth). In short, competitive markets in macroeconomic risk do not expunge credit cycles.[2] It is this behaviour that opens up scope for welfare-enhancing public policy, as the decentralized equilibrium is constrained inefficient: Society, though not individual entrepreneurs with access to aggregate risk markets, would prefer to go into recessions with a larger aggregate stock of inside wealth. And so, by subsidizing labour which is a complement of wealth, the authorities can move the economy closer to the constrained efficient outcome.

1.5 Financial contracts and factor prices

The equations in this section are derived from the optimal external financing contract, considering costly state verification, audit errors, and risk-averse agents. The firm/entrepreneur and the household agree on a contract that is incentive-compatible and penalizes the entrepreneur if found lying. This can be costly if a truthful report of low productivity is erroneously overturned. The contract also satisfies a participation constraint for the household. To attract household investment, entrepreneurs need to offer an expected return greater than r and cover expected audit costs. Due to the entrepreneur's risk aversion, a financial wedge emerges from the covariation of the entrepreneur's marginal utility and firm outcomes. This wedge increases with both firm-specific uncertainty and leverage. It affects the demand for labour and capital and also increases the equity risk premium. The derivations of (4,5,6) are contained in Appendix G.

Leverage is defined as the ratio of expected entrepreneurial production to the opportunity cost of entrepreneurial wealth.

$$l = \frac{y}{(1+r)q^e}, \tag{4}$$

where y denotes aggregate income, $y = zh^{\alpha}$. For an individual entrepreneur, an increase in either capital or labour inputs contributes to leverage. At the aggregate level, given fixed capital, leverage is increasing in aggregate labor employment. This

[2] In contrast with Di Tella (2017), productivity shocks are amplified through credit cycles in our model, as are interest rate shocks and cost-push shocks in the New Keynesian version of our model we present in Section 5.

risk-adjusted leverage measure captures how an increase in production increases the residual risk borne by entrepreneurs, since labor costs must be paid regardless of the idiosyncratic productivity outcome.

The equity risk premium is defined as the ratio of the expected return to entrepreneurial equity over the risk free rate. The equity risk premium is increasing in the leverage ratio l, a measure of the scarcity of entrepreneurial wealth, and a risk premium τ that compensates for the undiversifiable component of inside wealth.

$$\rho = 1 + l\tau \tag{5}$$

where $\tau = \dfrac{-\text{cov}_\theta\left(u^{e\prime}(c^e(\theta)), \theta\right)}{\mathbb{E}_\Theta[u^{e\prime}(c^e(\theta))]}$.

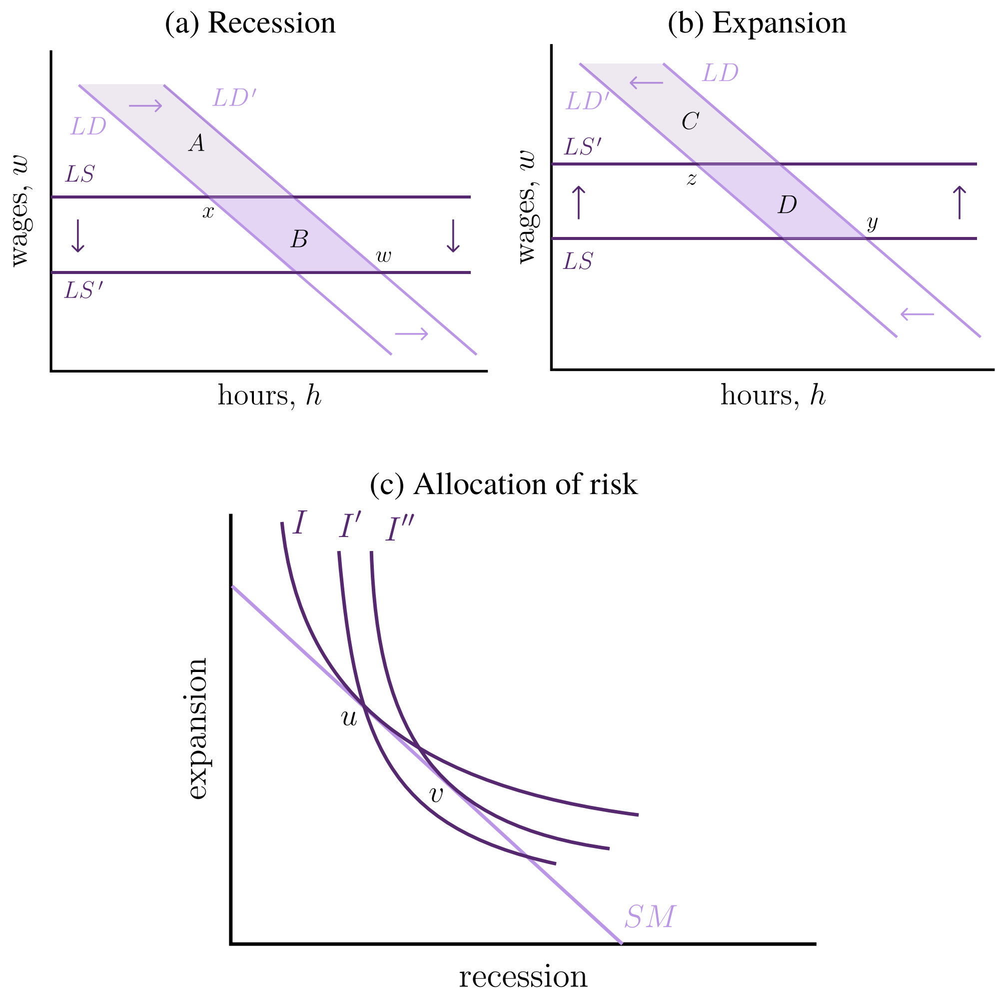
Wages differ from the expected marginal product of labour by the wedge of inefficiency τ, reflecting the marginal increase in residual risk borne by the entrepreneur resulting from an increase in labour hired,

$$w = \mathbb{E}[\theta] f'(h)(1 - \tau). \tag{6}$$

1.6 The effect of wage subsidies on entrepreneurial risk taking

To demonstrate the interaction between wage subsidies and the allocation of aggregate risk, consider Figure 2. It presents a partial equilibrium analysis of the effect of countercyclical wage subsidies on an individual firm's risk-taking. The firm starts at x, y, and u in panels (a), (b), and (c) respectively, taking wages as given. Consider a firm with access to aggregate risk insurance markets. The firm can insure against a recession or expansion in the next period. The labour supply schedule is initially given by LS. A firm may decrease its risk by buying insurance against a recession and selling it against an expansion. Thus, a small decrease in the firm's risk-taking before the realization of expansion or recession would shift the firm's labour demand from LD to LD'. This results in an increase in labour demand if the economy enters a recession and a decrease if it enters an expansion. The firm's allocation of risk is optimal before the realisation of the aggregate economic state if

Figure 2: Countercyclical wage subsidies and firm risk taking.



The diagrams represent the partial equilibrium effects on the labour market of the introduction of a countercyclical wage subsidy (LS to LS'), which motivates a decrease in firm risk taking (LD to LD'). The key insight is that countercyclical wage subsidies increase the value of firm wealth in recessions (panel a), making firms willing to purchase more recession insurance ex ante (panel c), moving from indifference curve I to I'.

they value areas $C + D = A$ after adjusting for their marginal rate of substitution across expansions and recessions.

Let the policymaker introduce a wage subsidy (tax) upon realisation of a recession (expansion), shifting labour supply from LS to LS'. A small decrease in the firm's risk taking increases surplus by A + B in recession, strictly greater than the decrease C in expansion. The value of wealth carried forward into the recession state has increased relative to the value of wealth carried forward into the expansion state, and the firm's indifference curves in the risk allocation market rotate. The firm chooses their risk allocation from their budget set, where the slope of the SM schedule reflects the relative market prices of Arrow securities contingent on the realisation of expansion and recession states reflectively. Ultimately, the firm reduces their risk taking, moving from u to v.

With the intuition of the model set out, we now set out the essential features of the financial friction and then turn to characterise optimal wage subsidy policies.

2 A quantitative model with imperfect state verification

Our choice of specific financial friction determines the quantitative relationships between the residual risk borne by entrepreneurs, their leverage, and idiosyncratic production risk. For our quantitative analysis, we assume that financial contracts are subject to a version of the imperfect state verification model following Duncan and Nolan (2019). Importantly, this environment generates tractable standard debt contracts as privately optimal, even when lenders can commit ex ante to ex post monitoring strategies, and even when the lender has access to lotteries, and without any limited liability constraints. This ensures that we can interpret welfare gains from policy as reductions in the efficiency costs of moral hazard, rather than reducing the private costs faced by agents constrained by a specific contractual form.[3]

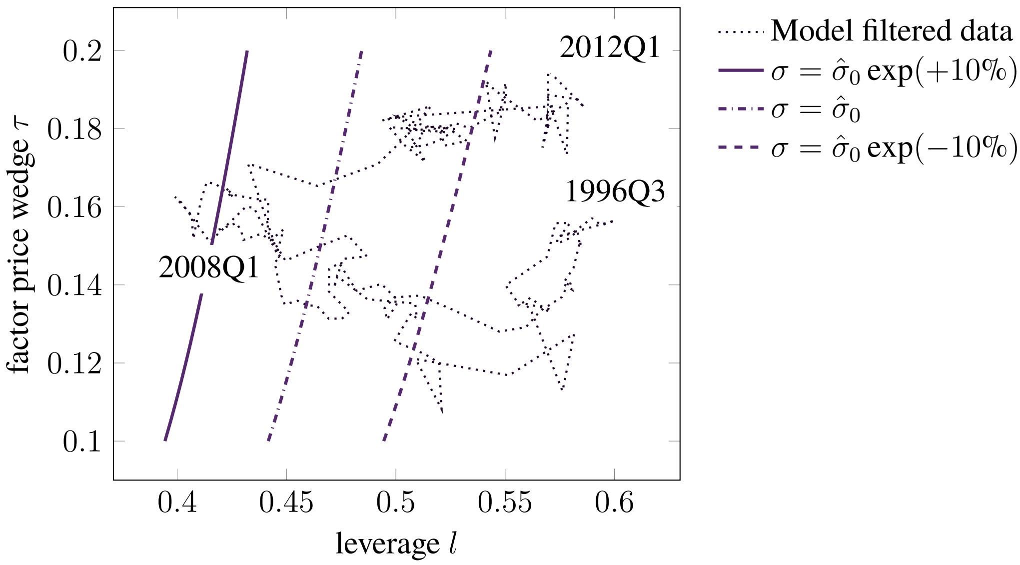
The upshot is that the contracts we study are privately optimal given the information and record keeping constraints that we impose. We do not require any reduced-form pledgeability constraints, which are unlikely to be robust to the policy

[3]Under perfect and deterministic audits, the environment would be that of Townsend (1979). Townsend noted that a better contract might employ a stochastic auditing schedule and this was confirmed by Border and Sobel (1987) and Mookherjee and Png (1989). These authors then showed that optimal contracts, rather than being debt, resemble equity finance—repayments are contingent on marginal fluctuations in income. However, with risk-averse entrepreneurs and audits that are imperfect and may wrongly indicate productivity draws, Duncan and Nolan (2019) show that once more debt contracts can be optimal. Equity-like contracts provide more insurance across productivity draws, but given imperfect audits may make a bad situation worse for a borrower with a genuine low productivity draw. Duncan and Nolan (2019) discuss in detail the literature and other requirements for the optimality of debt contracts in their imperfect audits csv model and fits the model to US data.

interventions that we consider. Specifically, our model predicts that policy interventions affect the implied pledgeable share of wealth. Clearly, these effects would be missing in models where the pledgeable share of wealth is exogenous.

This model provides a tractable convex mapping from leverage l and idiosyncratic risk σ to the factor price wedge τ. Figure 3 presents this relationship in the neighbourhood of the calibrated deterministic steady state, with model filtered values derived from US data. This function, derived in Appendix C, is described by Equation 7. In the neighbourhood of the deterministic steady state, the factor price wedge is sharply increasing in leverage and idiosyncratic risk.

Figure 3: Leverage and the factor price wedge



Notes: The dotted series presents the values for leverage and the factor price wedge that consistent with observed US data, filtered through equations (4) and (6). The solid, dashed, and dash-dotted schedules represent the relationship between leverage and the factor price wedge for different levels of idiosyncratic risk, given by Equation 7. All values are computed using a calibrated parameter vector. The parameter $\hat{\sigma}_0$ represents the steady state level of idiosyncratic risk.

$$l = \frac{(\overline{\pi} + \underline{\pi}\eta)\tau}{(\overline{\pi}\sigma - \tau)(\underline{\pi}\eta\sigma + \tau)}. \tag{7}$$

Here σ is proportional to the standard deviation of idiosyncratic productivity risk, and as before, $\underline{\pi}$ is the probability of a low draw of that risk, and $\overline{\pi}$ is the probability of a high draw. Finally, η is the probability of a Type-1 audit error; that is failing correctly to identify an entrepreneur who is truly low productivity. We allow σ to vary exogenously as a mean-preserving spread shock to idiosyncratic productivity risk.

Equation 7 is the only equilibrium condition in the model that depends on the assumption of imperfect state verification. Adapting the model for other agency problems (for example, hidden actions or costly state falsification) would mean replacing Equation 7, and the consumption growth equations (E1,E2) in the appendix required for welfare calculations. No further changes to model equations are necessary.

3 Optimal wage subsidy policy under log household utility

We first characterize optimal policy under log utility for households, which provides analytical tractability. This restrictive case yields closed-form solutions and clear intuition; we relax this assumption in our quantitative analysis (Section 4).

The policymaker weights the welfare of the representative household by μ, and the value of the representative entrepreneur by μ^e. The representative entrepreneur begins at time zero with wealth equal to mean entrepreneurial wealth. Social welfare v^s is defined as follows:[4]

$$v^s := \mu v + \mu^e v^e \tag{8}$$

We apply the Pareto weights that imply that the competitive equilibrium distribution of wealth is socially optimal at the economy's steady state. These weights can be interpreted as Negishi (1960) weights. Full details of the welfare calculations in our quantitative analysis can be found in Appendix E.

The representative household's consumption leisure trade-off becomes

$$\frac{-u_h}{u_c} = w(1+\varsigma) \tag{9}$$

[4]We assume that the policymaker is interested in the welfare of both groups in the economy. An alternative formulation would be to assume that the policymaker is only directly interested in the welfare of the working households, and to include a participation choice for entrepreneurs: the policymakers' weight on entrepreneurial welfare μ^e would then correspond to the Lagrange multiplier on the entrepreneurs' participation constraint.

The wage subsidy ς is funded by lump sum taxes, paid by the household sector. The household sector's budget constraint becomes

$$q' = (1+r)q + wh(1+\varsigma) - c - \int_{s\in S} p(s)x(s)ds + x(s') - T.$$

The policymaker faces the balanced budget constraint

$$T = wh\varsigma.$$

Proposition 1 *Assume the following utility functions:*

$$u(c,h) = \log c - \frac{h^{1+\psi}}{1+\psi}, \qquad u^e(c^e) = \log c^e.$$

a. *The socially optimal wage subsidy is described by Equation 10:*

$$\varsigma^* = \frac{\tau}{1-\tau}\left(1 - \left(\frac{l_0\rho - l\rho_0}{\rho - 1}\right)\frac{(1-\beta^e)}{l_0 - (1-\beta^e)\rho_0}\right). \tag{10}$$

b. *Output is completely stabilised in response to uncertainty shocks, and is proportional to total factor productivity.*

A proof of Proposition 1 can be found in Appendix B.

The implications of the optimal wage subsidy on equilibrium output are as follows. Under the competitive equilibrium in the absence of wage subsidy policy, real output follows

$$y = z\left(\frac{\alpha l(1-\tau)}{l - (1-\beta^e)\rho}\right)^{\frac{\alpha}{1+\psi}}. \tag{11}$$

Under the optimal wage subsidy, real output follows

$$y = z\left(\frac{\alpha l_0}{l_0 - (1-\beta^e)\rho_0}\right)^{\frac{\alpha}{1+\psi}} \tag{12}$$

Comparing Equations 11 and 12, real output is proportional to total factor productivity in both regimes, but only responds to fluctuations in leverage and risk under the competitive equilibrium. Under the optimal wage subsidy regime, output does not respond to uncertainty shocks.

The wage subsidy also reduces consumption risk over the cycle for households. As a consequence, households' demand for aggregate risk insurance falls, lowering the market price of insurance and further encouraging firms to purchase more aggregate risk insurance. This wealth effect of the wage subsidy for households further supports the prudential channel of wage subsidy policies.

Equivalently, the wage subsidy induces higher labour supply from households. It also induces higher inside wealth accumulation on the part of entrepreneurs (because they get the benefit of subsidised labour, a complement to their wealth stock). And so, entrepreneurs buy more aggregate (consumption) risk insurance for recession times than they otherwise would because the labour subsidy makes inside wealth more valuable during such times.

3.1 Time inconsistency of optimal subsidies

Optimal wage subsidies are time inconsistent (Proposition 2). In our model, wage subsidies impact the economy through two channels: (1) an ex-ante prudential effect (encouraging firms to accumulate wealth before downturns) and (2) an ex-post distortionary effect (reducing labor market efficiency during downturns). A discretionary policymaker in a downturn only experiences the costs of (2) without the benefits of (1), since firms' wealth accumulation decisions are already sunk.

Proposition 2 *Assume the risk free real interest rate r is fixed. Under discretion, the optimal wage subsidy is zero in all periods.*[5]

As is demonstrated in Section 5, commitment to wage subsidy policies is less important when the economy suffers from aggregate demand externalities. Under sticky price settings, the ex-post distortionary effects of wage subsidies can be a net-benefit. This helps to ameliorate the time-inconsistency problem. It is shown there that a wage subsidy policy response is part of the optimal policy response to a recessionary shock under discretion.

[5] Note that the assumption of a fixed risk free real interest rate has no effect on the optimal policy described in Proposition 1.

3.2 Financial amplification and risk aversion

Before turning to quantitative analysis of the benefits of wage subsidies, it is worth noting that while the assumption of log utility can be analytically convenient, it is restrictive. Specifically, when households are more risk averse than entrepreneurs—a natural assumption to make—leverage decreases in response to efficiency shocks, generating procyclical financial amplification. Equation 13 presents the derivative of current period leverage with respect to current period output. When household risk aversion is greater than under log utility ($\gamma > 1$), leverage is decreasing in current period output, generating a financial accelerator effect in response to TFP (and other) shocks.

$$\frac{dl}{dy} = -\frac{l}{y}\frac{(\gamma - 1)(l - (1 - \beta^e)\rho)}{l + (\gamma - 1)(1 - \beta^e)\rho}. \tag{13}$$

This financial amplification is an important feature of the model, and we allow for non-log household utility in the estimated non-linear model we use for quantitative analysis.

4 Optimal wage subsidy simple rule

Having characterised the optimal wage subsidy for the model under log utility, we now relax the assumption of log utility and turn to the analysis of a simple, optimised wage subsidy rule. This analysis is motivated by some of the same considerations that motivate the study of optimised simple rules in other areas of monetary and fiscal analysis (such as the vast literature on optimised 'Taylor Rules'). The idea is essentially that in practice policymakers often act like they condition instrument feedback control on a subset of widely observed variables. For our purposes, then, we propose the following wage subsidy simple rule

$$\varsigma = -\phi_\varsigma(y - y_0), \tag{14}$$

where y_0 is deterministic steady state output.

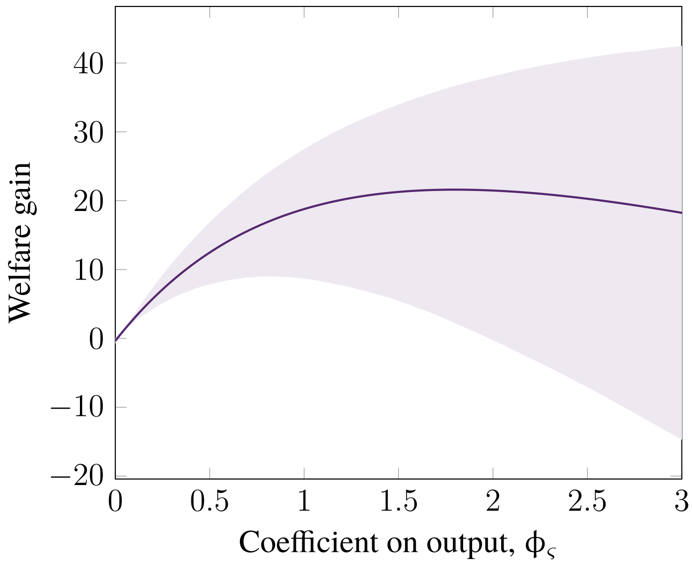
One way to think of this, is that rather than the policymaker observing and reacting to individual firm leverage or the financial wedge—firm specific or economy-wide—the policymaker is ‘hoping’ the rule-based subsidy conditioned merely on GDP is positively correlated with that fully optimal subsidy. This rule is also loosely motivated by attempts to support the labour market during the recent pandemic. Of course, most of the schemes introduced were not new. For instance, in New Zealand, the scheme was built on earlier schemes such as the Earthquake Job Loss Cover, a payment to labour which applied both to full time and part time workers. However, although many of the schemes were not new, the extent of their application was. And in some countries–like in the UK–the (furlough) scheme was in fact new. As we observed earlier, the schemes otherwise varied substantially across countries. The US for instance channelled more of its support for labour by extending unemployment insurance. Many other countries extended work-sharing schemes, or short-time working (Kurzarbeit) as in Germany and a number of other countries. Schemes also varied substantially across countries in terms of duration, generosity and many other aspects of design. As Giupponi et al. (2022) observe, schemes may be classified by their principal focus, either on insuring workers (as in the US) or insuring job matches as in the UK and many other OECD economies. However, as these authors also observe, in practice schemes were not so discontiguous. For instance, in the US context, recall unemployment insurance also has job match preservation features. In any event, the wage subsidy we model is inevitably a somewhat stylized version of those schemes actually implemented, but is clearly closer to schemes that emphasised job insurance (and which were in operation in many countries even before the pandemic).

Figure 4 shows the welfare gains resulting from wage subsidy simple rules, using our parameter estimates described in Appendix F. The persistence of the TFP shock affects the welfare assessment of subsidies. For our estimated parameters, a wage subsidy rule with coefficient $\phi_\varsigma = 1.8$ maximises expected welfare gains across posterior draws and simulated histories, reducing the expected welfare costs of business cycles by 20.5% relative to the counterfactual of no wage subsidy policy.

While modest wage subsidy simple rules generate sizeable welfare gains for all posterior parameter estimates, we find that the magnitude of welfare gains from wage subsidy simple rules is sensitive to the value of the persistence of TFP shocks. Figure 5 presents the persistence of TFP shocks, ρ_z, and the welfare gain from the associated optimal wage subsidy simple rule, for a sample of parameter draws from

Figure 4: Estimated mean welfare gain from optimal simple rule

(Percent decrease in business cycle welfare costs)



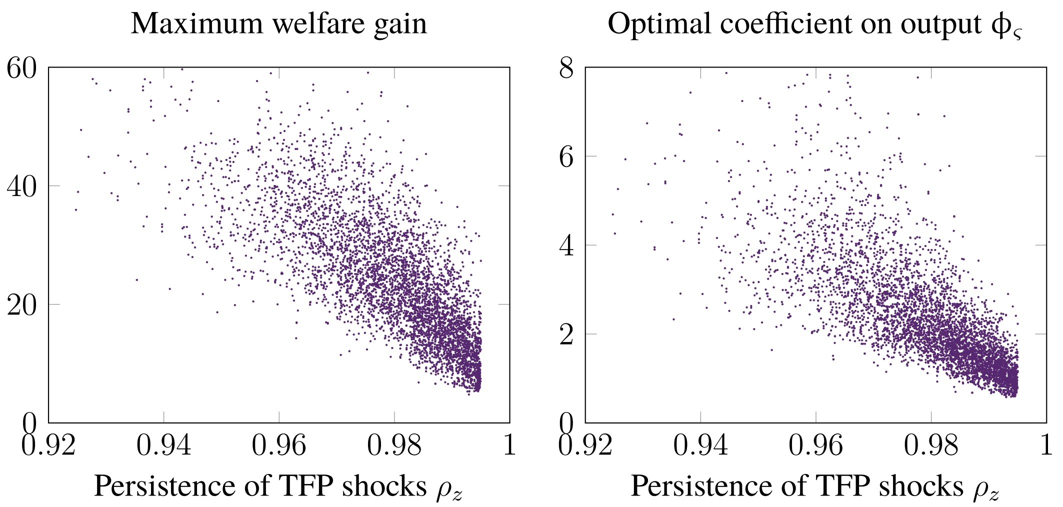
Notes: The welfare gain is reported as the reduction in the welfare costs of business cycles as a percentage of those welfare costs in the absence of policy intervention. Shaded area indicates 90% credible interval. Parameter values are estimated under the assumption that there is no wage subsidy policy.

the posterior. Posterior draws with lower values for the persistence of TFP shocks correspond to larger welfare gains from the fixed wage subsidy simple rule. When TFP shocks are highly persistent, the welfare costs of recessions are due more to the long tail of low productivity than to the financial acceleration of the recession itself. As a result, there is less scope for policy interventions to reduce the welfare costs of business cycles. In other words, subsidy schemes ought to be introduced quickly following a shock, but the value of the subsidy should be reduced more quickly than the shock's effects dissipate. It also seems that shocks to the financial friction also require more robust subsidies.

4.1 Crises and optimal simple rules

We turn now to the likely performance of the simple rule during a crisis. For this we need an example of a crisis. We draw parameters from the posterior parameter estimates and generate model simulated data. From the simulated data, we extract

Figure 5: Simple rule welfare gains and the persistence of TFP



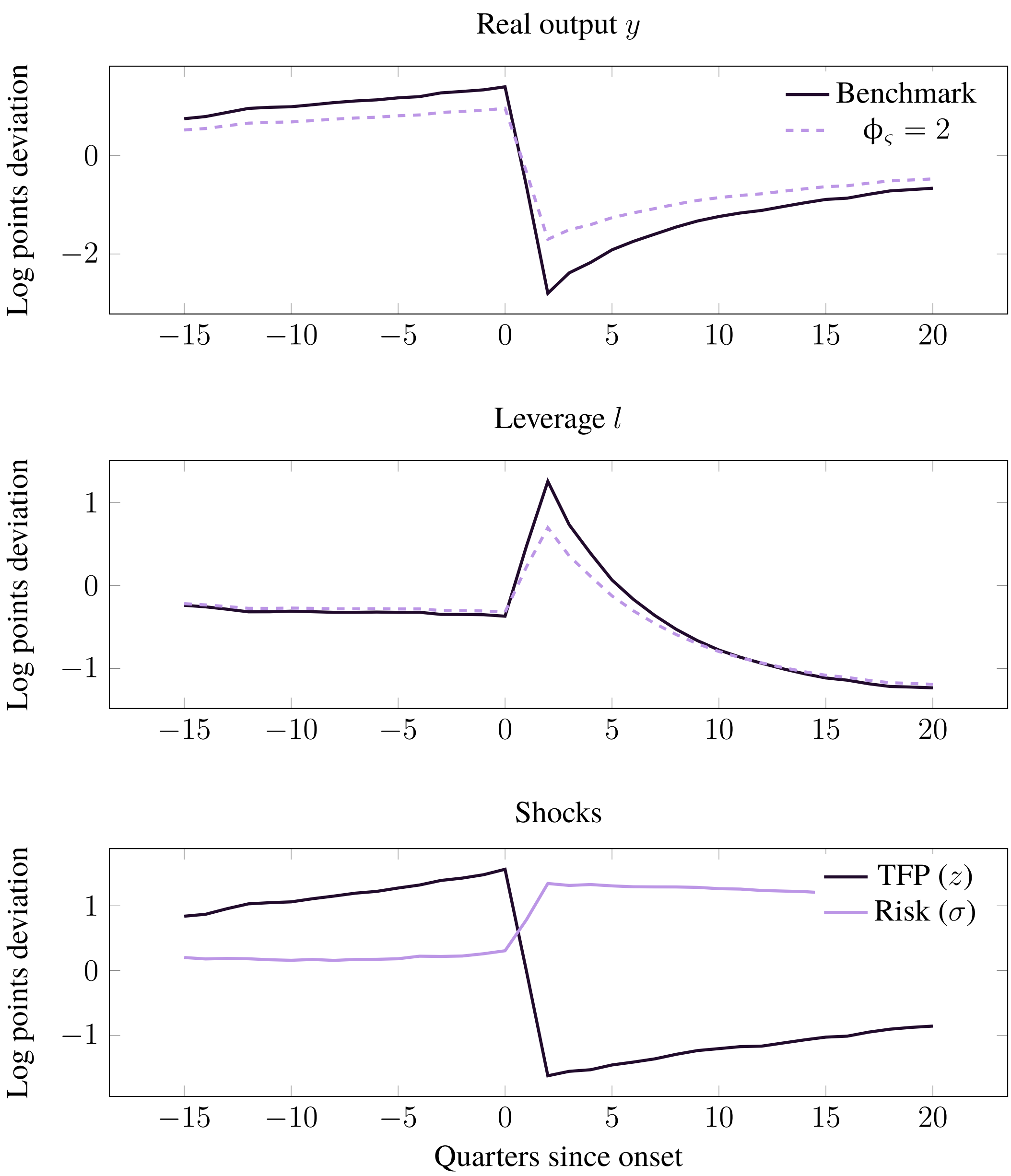
Notes: Each dot represents a sample drawn from the posterior of the estimated model. Given the associated vector of parameter values, the optimal wage subsidy simple rule is computed, and the for the optimal simple rule, the coefficient of the wage subsidy simple rule on output, and the associated welfare gain, are reported. The welfare gain is reported as the reduction in the welfare costs of business cycles as a percentage of those welfare costs in the absence of policy intervention.

episodes where real GDP falls by more than 4 percentage points over two quarters, reflecting the largest two quarter fall in de-trended real GDP during the US Great Recession.

The first panel of Figure 6 shows real output across the benchmark (i.e., no wage subsidy) and wage subsidy simple rule regimes. Under the wage subsidy simple rule regime, the typical depth of crises is smaller. When the coefficient of the simple rule on output is $\phi_\varsigma = 2$, the mean peak-to-trough fall in output is 28 percent smaller than in the no-policy counterfactual.

So, the wage subsidy simple rule would reduce the amplitude of business cycles in a frictionless economy, but to increase welfare in our model, the policy must reduce the cost of the frictions. The second panel shows the path of leverage under the two regimes. Despite increasing output relative to the benchmark in crisis episodes, the wage subsidy policies decrease (relative) leverage, and hence the resulting cost of financial frictions in crisis episodes. Leverage in our model is risk-based, and defined as the ratio of output to the opportunity cost of net worth. For wage subsidies

Figure 6: Economic crises with and without wage subsidy simple rules



Notes: All values are reported as mean log deviations from deterministic steady-state levels. Solid series represent the benchmark (no wage subsidy) regime, while dashed series represent an economy with a wage subsidy simple rule with a coefficient on output of 2. Crises are episodes where real output falls by at least 4 percentage points over two quarters in simulated data generated using parameters drawn from the model posterior. For each parameter vector, we then impose a common wage subsidy simple rule and generate new simulated data using the same shock sequences to generate the dashed series, extracting periods that were identified as crises under the benchmark regime.

to reduce leverage and increase output, they must have a precautionary impact on firms' risk taking in advance of the crisis episodes. This is the mechanism through which wage subsidy policies not only reduce the amplitude of business cycles, but also increase welfare in the model.

Crises also illustrate the limitations of the simple-rule approach to wage subsidy policies. In the absence of wage subsidies, the labour wedge increases by 4.47 percentage points as the economy moves from peak to trough. With the wage subsidy simple rule, the labour wedge decreases by 0.87 percentage points: the endogenous factor wedge τ increases by 3.48 percentage points due to the increases in leverage and risk, but the increase in the factor wedge is offset by the wage subsidy, which reduces the total labour wedge by 4.22 percentage points.[6]

Ten quarters after the onset of the crisis, the labour wedge under the wage subsidy simple rule is 2.04 percentage points smaller than pre-crisis levels, as the wage subsidy has remained high despite the fall in leverage and the factor wedge. This is likely to be inefficient: at this point, leverage has returned to near pre-crisis levels under both policy regimes. Output remains well below trend, but low output is not inconsistent with efficient allocations given the persistent effects of the exogenous shocks. At this point, the wage subsidy resulting from the simple rule is likely to be doing more harm than good.

The third panel of Figure 6 shows the typical paths of exogenous shocks around crisis periods. Crisis episodes are typically generated by a combination large negative technology shocks, contractionary risk shocks, and below steady state leverage.

4.2 Distorted labour markets: Dixit's critique

So far, our analysis assumes undistorted labour markets where wage subsidies generate only second-order costs. Labour markets are typically highly distorted; the introduction of additional wage subsidies and taxes generates first order rather than second order costs. This can potentially overturn the optimality of interventions targeted at alleviating moral hazard costs (Dixit, 2003).

[6] Given wage subsidy ς, the labour wedge becomes

$$1 - \frac{-u_h}{u_c \mathbb{E}[\theta] f'(h)} = \tau - (1-\tau)\varsigma$$

We add a constant 40% labour tax to the baseline model and simulate the model to find the effects of optimal policy simple rules from this new distorted base. Tax revenues are rebated lump sum to households. That 'wedge' is a little higher than the average tax wedge on the average OECD worker[7] and is in the ballpark of some of the estimates for the US over a long run of data calculated by Mulligan (2002).[8] It is important to point out that in Mulligan's data the wedge is not solely the result of taxes, a point we return to presently. We find that after introducing the 40% labour 'tax', the optimal coefficient of the wage subsidy simple rule on output, ϕ_ς, increases from 1.80 to 3.17. The resulting expected welfare gain increases from 20.5% to 30.3% of counterfactual business cycle welfare costs.

The invariance of optimal wage subsidy policy to static tax distortions is perhaps not surprising; the relevant margin for our wage subsidy policy is not the steady state labour margin, but instead the dynamic allocation of labour across business cycle outturns. In the following section, we introduce a time-varying labour market distortion in the form of New Keynesian product market nominal rigidities.

5 Optimal wage subsidies and time varying markups

The flexible-price model demonstrates that prudential wage subsidies improve welfare but face a severe time-inconsistency problem. We now introduce nominal rigidities for two reasons: First, to verify our results are robust to realistic time-varying labor market distortions (Dixit's critique). Second, and more importantly, to show that aggregate demand externalities partially resolve the time-inconsistency problem, making commitment less critical for policy effectiveness. In this section we present a linearized, small scale New Keynesian version of our model. The full derivation of the model can be found in Duncan, Mainente, and Nolan (2023).

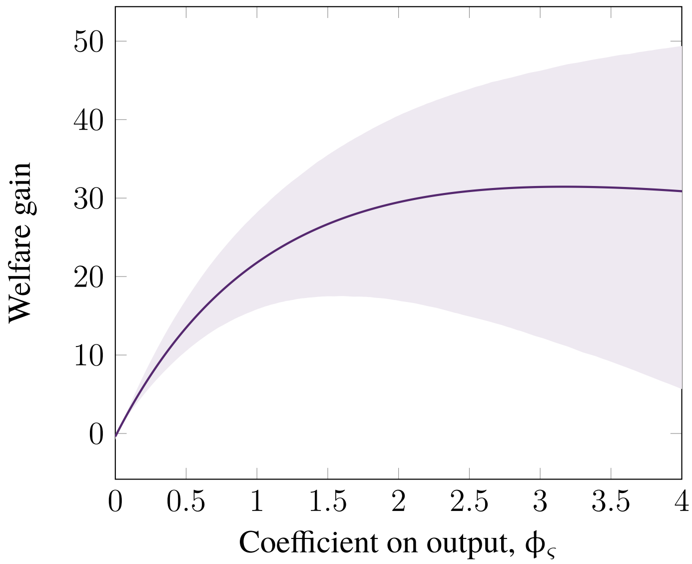
Countercyclical wage subsidy policies remain welfare improving in the presence of time varying product market distortions that generate cyclical fluctuations in the labour wedge of inefficiency (Proposition 3). Under jointly optimal monetary and wage subsidy policies, inflation is stabilised for uncertainty shocks, and wage subsidies are phased out quickly after their introduction (Section 5.2). Moreover, in the

[7] See the OECD, Taxing Wages 2022.

[8] A Century of Labor-Leisure Distortions, NBER Working Paper 8774.

Figure 7: Estimated mean welfare gain from optimal simple wage subsidy rule

(Constant labour tax of 40%)



Notes: The welfare gain is reported as the reduction in the welfare costs of business cycles as a percentage of those welfare costs in the absence of policy intervention. Shaded area indicates 90% credible interval. Parameter values are estimated under the assumption that there is no wage subsidy policy.

presence of aggregate demand externalities, optimal wage subsidy policies are not as severely affected by the time inconsistency problem as in the flexible price economy (Section 5.3. The flexible price case was analysed in Section 3.1). The proofs of all propositions in this Section are contained in Appendix H.

The New Keynesian version of our model is characterised by three principle equations (15, 16, and 17).

The IS curve

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\gamma} r_t + \omega(1-\varphi)\mathbb{E}_t[\Delta l_{t+1}] - \omega\varphi\mathbb{E}_t[\Delta\sigma_{t+1}], \tag{15}$$

the Phillips curve

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \lambda \mathrm{pp}_t, \tag{16}$$

and the Leverage curve

$$\Delta l_t = \tfrac{1}{1+\gamma\omega(1-\varphi)} \left(-\varphi(l_{t-1} + \sigma_{t-1}) + \gamma\omega\varphi\Delta\sigma_t - (\gamma - 1)\Delta x_t\right), \qquad (17)$$

where $\frac{\omega}{1+\omega}$ is the steady state entrepreneurial consumption share, and φ is the elasticity of the equity risk premium with respect to leverage and risk. In this Section, output y, leverage l, uncertainty σ, technology z, and producer prices pp are expressed in terms of their respective log deviations from deterministic steady state levels. The real interest rate is $r_t := i_t - \mathbb{E}_t[\pi_{t+1}]$ where nominal interest rate is i and inflation π. The factor wedge τ, and the wage subsidy ς are expressed in terms of their respective deviations from deterministic steady state levels. The operator Δ takes the growth rate of its argument, $\Delta l_t = l_t - l_{t-1}$.

Our IS curve differs from the benchmark New Keynesian IS curve as a result of the time varying distribution of consumption between entrepreneurs and the representative household.[9] Our IS curve is derived from the representative household's optimal consumption savings plan. Expected household consumption growth depends on expected output growth but also current and expected future leverage and uncertainty: an increase in leverage or uncertainty reduces the expected growth of the households' consumption share of total output.

Retailers purchase the wholesale good from entrepreneurs in competitive markets. From this wholesale good the retailers produce a differentiated retail good. The retail goods are sold in monopolistically competitive markets subject to Calvo pricing frictions. The producer prices paid by the retailers for the wholesale good are equal to the total marginal costs of entrepreneurial production. These costs comprise marginal labour costs and the risk premium that constitutes the labour wedge of inefficiency τ. Marginal labour costs are the sum of traditional New Keynesian marginal costs (increasing in the output gap), a wealth effect on labour supply resulting from fluctuations in the distribution of consumption (when the household's consumption share is high, wages demanded are higher for every level of the output gap), and the wage subsidy policy instrument ς.

[9] For an exposition of the benchmark New Keynesian model, refer to Galí (2008, Chapter 3).

$$\text{pp}_t = \underbrace{\left(\gamma + \tfrac{\varphi+\alpha}{1-\alpha}\right) x_t - \tfrac{1+\varphi}{1-\alpha}\, z_t}_{\text{benchmark model marginal costs}} + \underbrace{\gamma\omega(1-\varphi)l_t - \gamma\omega\varphi\sigma_t}_{\text{consumption inequality wealth effect}} + \underbrace{\tau_t}_{\text{labour wedge}} - \underbrace{\varsigma_t}_{\text{wage subsidy}} . \tag{18}$$

The Leverage curve (17) is unchanged from our benchmark flexible price model up to log-linearisation. The policymaker's welfare function (provided in Appendix H) differs from the flexible price welfare function as a result of the social costs of price dispersion during periods of abnormally high or low inflation.

5.1 Optimal wage subsidy policy taking the monetary policy rule as given

In this subsection we close the model with a Taylor-type monetary policy rule that responds to current inflation. We also impose log-utility for households to simplify exposition.

Assumption 1 *Worker households have log utility, $\gamma = 1$. Interest rates follow the rule: $i = \phi_\pi \pi_t$*

Proposition 3 *Given Assumption 1,*

(i) optimal wage subsidies increase in response to cost-push components of fluctuations in leverage and uncertainty, increase in response to contractionary supply shocks, and increase in response to expansionary aggregate demand components of fluctuations in leverage and uncertainty.

(ii) Optimal Wage subsidies are temporary, and are withdrawn more quickly than the underlying shocks dissipate.

(iii) The optimal wage subsidy policy is characterised by Equation 20.

Combining the equilibrium conditions yields the following characterisation of optimal policy. First, the optimal path of inflation follows

$$\pi_t = \mu\pi_{t-1} + s_t \tag{19}$$

$$\varsigma_t = -\mu \left(\lambda \chi \frac{\phi_\pi - \mu}{1-\mu} + (1-\beta\mu) \right) \pi_{t-1} + u_t \tag{20}$$

where s_t, u_t are functions of current period shocks and leverage (which is independent of policy under log utility), increasing in the cost-push and aggregate demand components of those shocks. The endogeneous persistence of inflation and output under optimal wage subsidy policy is captured by μ, a real-valued composite parameter in $(0, 1)$. Full characterisations of s_t, u_t, μ are provided in the Appendix H.

Leverage and risk both increase marginal costs of production in our model, similar to New Keynesian cost-push shocks. Following a standard New Keynesian cost-push shock, an optimal wage subsidy policy would completely offset the contribution of the shock to marginal costs, restoring the first best allocation. The optimal wage subsidy policy would then be withdrawn as the shock dissipates. In contrast with standard New Keynesian cost-push shocks, wage subsidies in our model cannot restore first best efficient allocations in response to uncertainty and technology shocks in our model. Welfare costs of uncertainty shocks and leverage dynamics remain even when the paths of output, real interest rates and inflation are stabilised. Optimal wage subsidies are withdrawn more quickly than the underlying shocks dissipate, as their primary role is prudential, and is most impactful in response to the unpredictable component of the shock on impact, rather than the predictible persistent component of the shock over time.

Why is this prudential? Start with the leverage updating relationship (17), adjusted for log utility:

$$\Delta l_t = \tfrac{1}{1+\omega(1-\varphi)} \left(-\varphi(l_{t-1} + \sigma_{t-1}) + \omega\varphi\Delta\sigma_t \right) \tag{21}$$

It is apparent that leverage is invariant to current period output and current period wage subsidies. Wage subsidies increase output, so for leverage to be invariant to wage subsidies, it must be the case that entrepreneurs bring more wealth into periods where wages are subsidised; conditional wage subsidies must generate an ex ante portfolio reallocation. If wage subsidies are introduced during downturns, then wage subsidies will motivate precautionary behaviour, where entrepreneurs bring more wealth into downturns.

One can see what happens when this precautionary channel is closed off. This is revealed in the leverage updating rule when only trade in non-contingent nominal bonds is permitted and aggregate risk markets are shut down. The resulting leverage updating relationship is

$$\Delta l_t = -\varphi l_{t-1} - \varphi \sigma_{t-1} - (i_{t-1} - \pi_t) + \Delta y_t \tag{22}$$

Comparing 22 with 21 shows the importance of the prudential effect of wage subsidy policies. Under closed aggregate risk markets (22), an increase in current period output (relative to current period inflation) increases current period leverage, increasing current period marginal costs and reducing the effectiveness of the wage-subsidy policy.

5.2 Joint optimal wage subsidy and monetary policy

Under log utility, optimal wage subsidy and monetary policies eliminate fluctuations in output and producer prices resulting from uncertainty shocks, similar to the optimal response to markup shocks in the standard New Keynesian model.

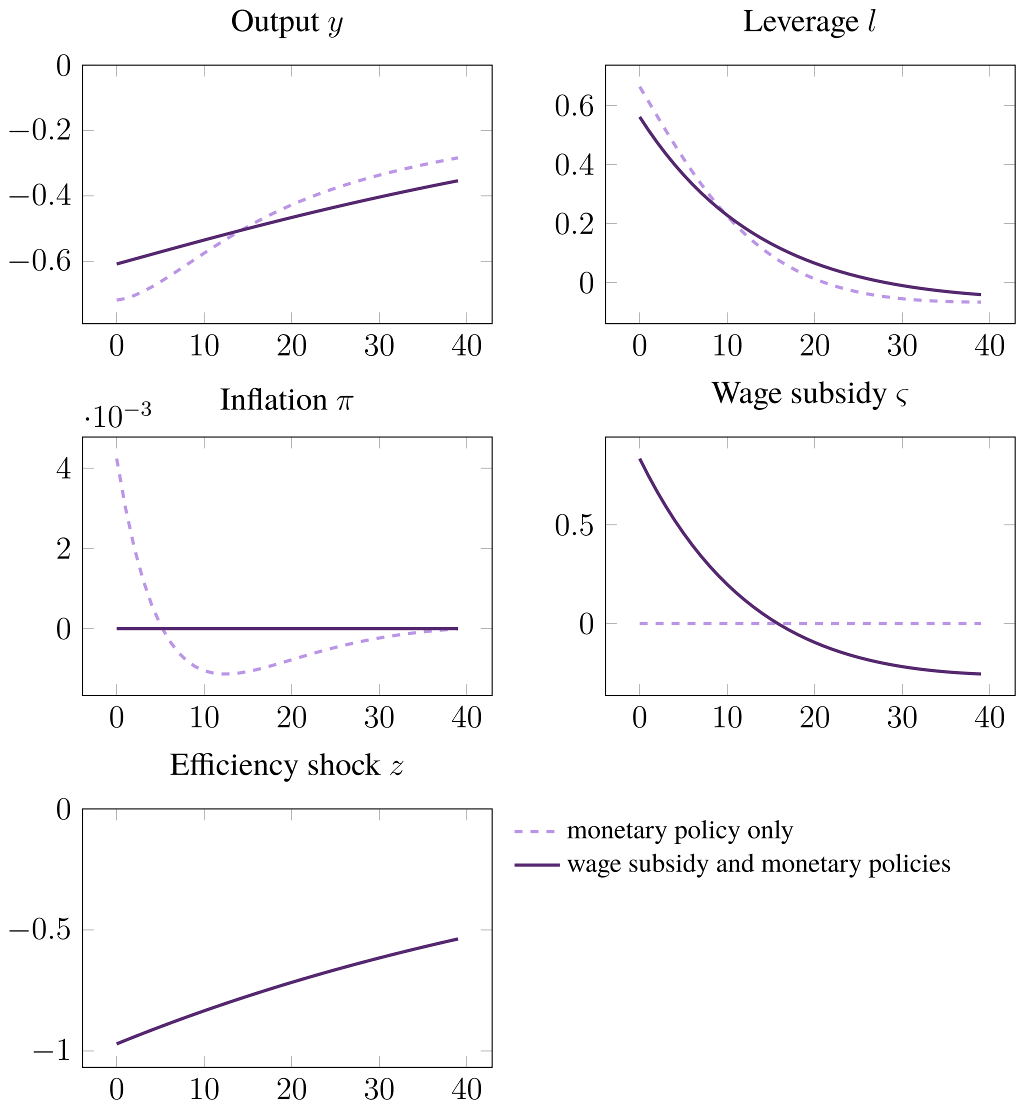
When households are more risk averse, ($\gamma > 1$) optimal policy no longer eliminates fluctuations in output resulting from uncertainty shocks.

Remark 1 *Under joint optimal monetary and wage subsidy policy, the optimal path of inflation is zero in all periods* $\pi_t = 0\ \forall t$.

The proofs of Remark 1 can be found in Appendix H. It is straightforward to show that Remark 1 also holds in response to traditional New Keynesian markup shocks when the policymaker has access to wage subsidies.[10] The convexity of monitoring costs generates a wedge between the social marginal costs of leverage and the private marginal costs of leverage. Intuitively, when the convexity of monitoring costs is high, fluctuations in leverage have large social costs, and optimal wage subsidy policy seeks to smooth the path of leverage. By inspection of 17, we can

Figure 8: Optimal monetary and wage subsidy policies under commitment

Optimal monetary policy and jointly optimal monetary and wage subsidy policy responses to recessionary 1sd efficiency shock.



Values are expressed in log point deviations from steady state. The estimated model is closed with the Taylor-type interest rate rule $i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_y (y_t - \varrho z_t)) + \varepsilon_{it}$, where $\varrho := \frac{\chi}{\chi + \gamma - 1}$. Model estimation details can be found in Appendix F.

see that the path of leverage is smoother if output falls when leverage is high.

Figure 8 presents the economy's response to a recessionary efficiency shock in the model under optimal monetary policy and under optimal combined monetary and wage subsidy policy. Under jointly optimal policy, wage subsidies increase sharply in response to the shock, damping the output, leverage, and inflation responses to the shock. Optimal wage subsidies decay more quickly than output or the efficiency shock itself.

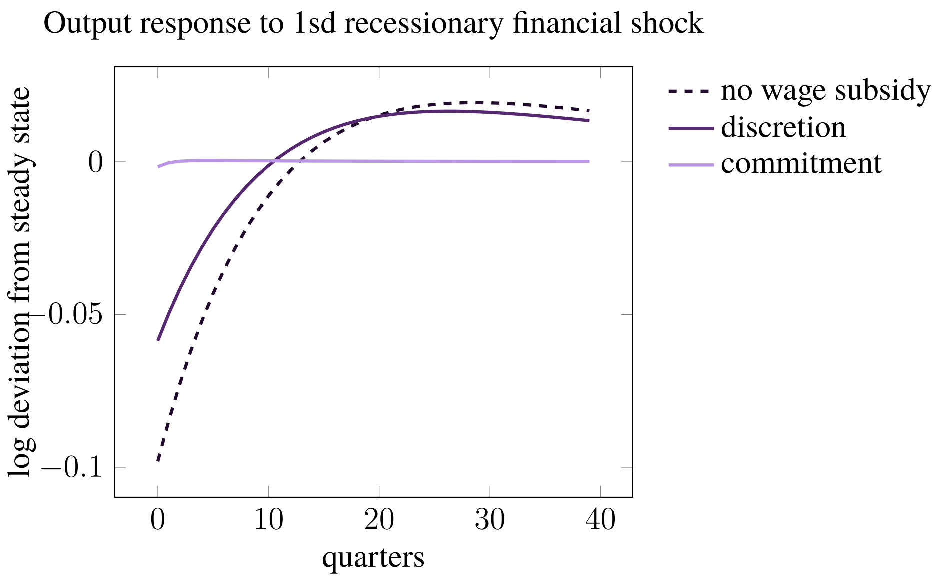
5.3 Commitment versus discretion under log utility ($\gamma = 1$)

In the flexible price model, we saw that a fiscal policymaker optimising under discretion would not levy countercyclical wage subsidies (Section 3.1, Proposition 2). Ex post, any wage subsidy in the flexible price model introduces a distortion into the labour market, reducing welfare. The welfare gains from countercyclical wage subsidies in the flexible price model result from the precautionary behaviour of entrepreneurs who anticipate the introduction of future wage subsidies in downturns. Conditional upon that precautionary behaviour ex ante from entrepreneurs, the policymaker has no incentive to introduce the wage subsidy when the downturn arrives. In equilibrium, there is no precautionary behaviour, and there are no wage subsidies. In the presence of nominal rigidities, this laissez-faire result is overturned. Aggregate demand externalities motivate a policymaker under discretion to stimulate the economy in downturns, including with their wage subsidy instrument. Anticipating the discretionary policymaker's incentive to stimulate the economy with wage subsidies, entrepreneurs will exercise ex ante precaution, generating a prudential benefit.

In the presence of aggregate demand externalities, optimal wage subsidies under discretion still respond to shocks. High leverage and uncertainty generate cost pressure through the Phillips curve, and optimal wage subsidies under discretion respond to this cost pressure. Figure 9 presents an example showing the response of output to a recessionary uncertainty shock in the absence of wage subsidy policy, under a discretion regime and a commitment regime. Under commitment, the

[10]Another similarity with markup shocks is that in the absence of wage subsidies, the optimal monetary policy allows inflation to deviate from target in response to uncertainty shocks.

Figure 9: Optimal wage subsidy policies under commitment and discretion



Notes: Values are expressed in log point deviations from steady state. The model is re-estimated to produce this plot with two additional parameter restrictions: log household utility ($\gamma = 1$) and the simplified Taylor-type interest rate rule $i_t = \phi_\pi \pi_t$.

optimal policy eliminates most of the volatility of output, and hours, in response to the shock, a small departure from the flexible price model with log utility where output is completely stabilised under commitment in response to uncertainty shocks (Proposition 1.b).[11] Under discretion, the optimal policy still dampens the volatility of output, in contrast with the flexible price case. The optimality of countercyclical wage subsidy policy under discretion is largely due to the coincidence of aggregate demand externalities and the leverage externality. The wage subsidy policymaker's efforts to reduce the cost-push consequences of high leverage have an unintended prudential effect. In equilibrium, discretionary countercyclical wage subsidies motivate a precautionary response from firms ex ante, who bring more equity into the recession than in the absence of the wage subsidies.

[11]This should not be confused with restoring first best efficient allocations—the distribution of consumption remains volatile, and the entrepreneur still bears welfare costs of higher idiosyncratic consumption risk.

6 Discussion

When governments and policymakers intervene in the economy to affect aggregate outcomes or support particular sectors or agents, it may be in crisis conditions when thoughts of moral hazard are set to one side. As Goodhart noted:

> The time to worry about moral hazard is in the boom. The first priority is to get out of the present hole. Worrying about moral hazard in [crisis] circumstances is rather like refusing to sell fire insurance just after the Great Fire of London for fear of adversely affecting future behaviour.[12]

We would take this further, and argue that the time to worry about the stimulus is in the boom. Well planned and predictable fiscal interventions can be designed to mitigate moral hazard problems while providing effective stimulus in crises.

In this paper, we add to the contributions of Milne (2020), Romer and Romer (2022), Woodford (2022), Guerrieri et al. (2022) and others. These papers study the roles of social insurance and stimulus in addressing crises whilst we shift the focus to the concern alluded to, but unaddressed, by Goodhart's observation. Our analysis yields three key insights: First, fiscal stimulus can increase welfare by mitigating moral hazard, even without aggregate demand externalities. Second, optimal policy is time-inconsistent. Third, aggregate demand externalities partially resolve this inconsistency.

References

Bank of England. Minutes of the Bank of England's Court of Directors, april 1857 to april 1858, 1858. URL `https://www.bankofengland.co.uk/sitemap/minutes`.

Robert J. Barro and David B. Gordon. Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics*, 12(1):101–121, 1983. ISSN 0304-3932. doi: https://doi.org/10.1016/0304-3932(83)90051-X.

[12]Not the Time to Worry about Moral Hazard by Charles Goodhart, *Financial Times*, September 18 2008.

Ben S. Bernanke, Mark L. Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1, Part C, chapter 21, pages 1341–1393. Elsevier, 1 edition, 1999.

Kim C Border and Joel Sobel. Samurai accountant: A theory of auditing and plunder. *Review of Economic Studies*, 54(4):525–40, October 1987.

Charles T. Carlstrom, Timothy S. Fuerst, and Matthius Paustian. Optimal Contracts, Aggregate Risk, and the Financial Accelerator. Working Paper 1420, Federal Reserve Bank of Cleveland, October 2014.

Sebastian Di Tella. Uncertainty shocks and balance sheet recessions. *Journal of Political Economy*, 125(6):2038–2081, 2017. doi: 10.1086/694290.

Avinash Dixit. Whither Greenwald-Stiglitz?, October 2003. Presentation at Joseph Stiglitz's 60th birthday conference, Columbia University.

Alfred Duncan and Charles Nolan. Disputes, debt and equity. *Theoretical Economics*, 14(3), September 2019.

Alfred Duncan, João Pedro De Camargo Mainente, and Charles Nolan. Optimal Macro-Financial Policies in a New Keynesian Model with Privately Optimal Risk Taking, 2023.

International Monetary Fund. Policy tracker, 2021. URL `https://www.imf.org/en/Topics/imf-and-covid19/Policy-Responses-to-COVID-19`. Accessed: 2021-02-09.

Jordi Galí. *Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Introductory Chapters. Princeton University Press, 2008.

Giulia Giupponi, Camille Landais, and Alice Lapeyre. Should we insure workers or jobs during recessions? *Journal of Economic Perspectives*, 36(2):29–54, May 2022. doi: 10.1257/jep.36.2.29.

Veronica Guerrieri, Guido Lorenzoni, Ludwig Straub, and Iván Werning. Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages? *American Economic Review*, 112(5):1437–1474, May 2022. doi: 10.1257/aer.20201063.

Nobuhiro Kiyotaki and John Moore. Credit Cycles. *Journal of Political Economy*, 105(2):211–48, April 1997.

Arvind Krishnamurthy. Collateral constraints and the amplification mechanism. *Journal of Economic Theory*, 111(2):277–292, 2003.

Finn E. Kydland and Edward C. Prescott. Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, 85(3):473–491, 1977. ISSN 00223808, 1537534X.

Alistair Milne. A critical COVID-19 economic policy tool: Retrospective insurance, March 2020. URL `https://dx.doi.org/10.2139/ssrn.3558667`.

Dilip Mookherjee and Ivan Png. Optimal auditing, insurance, and redistribution. *The Quarterly Journal of Economics*, 104(2):399–415, May 1989.

Takashi Negishi. Welfare economics and existence of an equilibrium for a competitive economy. *Metroeconomica*, 12(2-3):92–97, 1960. doi: 10.1111/j.1467-999X.1960.tb00275.x.

Kalin Nikolov. Collateral amplification under complete markets. *Journal of Economic Dynamics and Control*, 45(C):80–93, 2014.

International Labour Organization. Temporary wage subsidies: Country examples, an appendix to the ilo factsheet: Temporary wage subsidies, May 2020.

Christina D. Romer and David H. Romer. A social insurance perspective on pandemic fiscal policy: Implications for unemployment insurance and hazard pay. *The Journal of Economic Perspectives*, 36(2):3–28, 2022. ISSN 08953309, 19447965.

Robert M. Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293, October 1979.

UK House of Commons. Report from the Select Committee on the Bank Acts, 1858.

Michael Woodford. Effective Demand Failures and the Limits of Monetary Stabilization Policy. *American Economic Review*, 112(5):1475–1521, May 2022. doi: 10.1257/aer.20201529.

A Aggregate risk sharing in the competitive equilibrium

To derive the aggregate risk sharing condition in the main text we first set out the optimal savings decision of the two sets of agents, entrepreneurs (denoted with superscript e) and households. There is a unit measure of each agent. The aggregate state is denoted s and θ denotes the entrepreneur's idiosyncratic state. Agents in the model can buy and sell aggregate-state (Arrow-Debreu) risk assets as well as invest in risky, idiosyncratic and risk-free assets. Thus, one obtains for individual entrepreneur j:

$$\beta^e E_\theta u^{e\prime}(c^e_{j,t+1}) = u^{e\prime}(c^e_{j,t}) p_t(s_{t+1})$$
$$\beta u'(c_{t+1}) = u'(c_t) p_t(s_{t+1}).$$

With a slight abuse of notation, E_θ denotes solely the expectation over the realisation of idiosyncratic states, in order to be explicit about the sources of uncertainty. Lagging by one period the foregoing expressions, since time t idiosyncratic risk is intra-period and is not resolved (unlike s_t) before other time t decisions have to be made (like hiring labour, investing in risky idiosyncratic production), it follows from rearrangement that

$$\beta^e \left(\frac{E_\theta u^{e\prime}(c^e_{j,t})}{u^{e\prime}(c^e_{j,t-1})} \right) = \beta \left(\frac{u'(c_t)}{u'(c_{t-1})} \right).$$

Within period t, at the time of negotiating external finance, non-contingent savings and loans are the opportunity cost of state-contingent loans, therefore

$$(1 + r_{t+1}) E_\theta u^{e\prime}(c^e_{j,t+1}) = E_\theta u^{e\prime}(c^e_{j,t+1}) R_{t+1}(\theta, s_{t+1}).$$

Note that given intertemporal homotheticity and scalable technologies, the function $R_{t+1}(\theta, s_{t+1})$ is independent of the past history of individual entrepreneur j. So, we re-write the risk sharing relationship above as

$$\beta^e \left(\frac{E_\theta u^{e\prime}(c^e_{j,t+1}) R_{t+1}(\theta, s_{t+1})}{u^{e\prime}(c^e_{j,t-1})} \right) = \beta (1 + r_{t+1}) \left(\frac{u'(c_t)}{u'(c_{t-1})} \right).$$

We want to express the above risk sharing relationship in terms of aggregate levels of entrepreneurial consumption, rather than the paths of individual entrepreneurs' consumption. Consider the product $E_\theta[u^{e\prime}(c^e_{j,t}) R_t(\theta, s_t)]$.

Under log utility

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = E_\theta\left[\frac{R_t(\theta, s_t)}{c^e_{j,t}}\right].$$

Consumption decisions in period t are made after the individual realisation of R_t, and at the time of consumption, the total wealth available to individual entrepreneur is proportionate to their realisation of R_t. It follows, given intertemporal homotheticity, that $c^e_{j,t} = kR_t(\theta, s_t)$ for some $k \perp \theta$. The term k is not necessarily constant, but is independent to the time t realisation of θ. It follows that

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = \frac{1}{k}.$$

Now, let $\bar{c}^e_{j,t} = E_\theta[c^e_{j,t}]$, $\bar{R}_t = E_\theta[R_t(\theta, s_t)]$. By construction of k,

$$\bar{c}^e_{j,t} = k\bar{R}_t.$$

Therefore we have

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = \frac{\bar{R}_t}{\bar{c}^e_{j,t}}.$$

Under log utility,

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = u^{e\prime}(\bar{c}^e_{j,t})\bar{R}_t.$$

To summarise, under log utility, the expected product of marginal utility and gross returns to entrepreneurial wealth is equal to the product of the marginal utility of expected consumption and the expected gross return to entrepreneurial wealth.

Now, we set the index j to denote the entrepreneur who exits period $t-1$ with consumption equal to the mean consumption across all entrepreneurs, which we denote $\bar{c}^e_{t-1}$. It follows that

$$\beta^e\left(\frac{u^{e\prime}(\bar{c}^e_t)}{u^{e\prime}(\bar{c}^e_{t-1})}\right)E_\theta\left[\frac{R_t(\theta, s_t)}{1+r_t}\right] = \left(\frac{u'(c_t)}{u'(c_{t-1})}\right).$$

Re-writing the previous expression with $\rho \equiv \frac{E_\theta R(\theta,s)}{(1+r)}$, $\lambda \equiv u'(c)/u^{e\prime}(\bar{c}^e)$,

$$\lambda_t = \lambda_{t-1}\frac{\beta^e}{\beta}\rho_t,$$

where λ_t is measurable in the time t information set. This is the aggregate risk-sharing result of the main text.

Under perfect risk sharing for individual specific risks, the equity risk premium would disappear ($\rho = 1$), and we would be left with the traditional risk sharing relationship $\lambda_t = \lambda_{t-1}\frac{\beta^e}{\beta}$. The equity risk premium reflects the wedge between entrepreneur's expected marginal utility of consumption and the marginal utility of expected entrepreneurial consumption. An important consequence of imperfect idiosyncratic risk sharing is that it might not be possible to test aggregate risk sharing from aggregate consumption data alone.

B Policymaker's problem (Proof of Proposition 1)

In order to solve for the optimal wage subsidy, we present a minimal set of equilibrium conditions from the competitive economy, excluding the household's labour supply condition.

We solve for the policymaker's optimal labour supply, and use this solution to derive the wage subsidy that supports this labour supply. While non-zero wage subsidies may require lump sum transfers to finance, these transfers are ultimately traded away in competitive markets for aggregate risk, and do not appear in our policymaker's programme.

Equilibrium conditions

The following equilibrium and market clearing conditions are constraints faced by the policymaker.

$$zh^\alpha \geq c + c^e \tag{Λ_1}$$

$$lc^e = zh^\alpha(1-\beta^e)\mathcal{R}(l,\sigma) \tag{Λ_2}$$

$$\lambda' = c^{-\gamma}c^e \tag{Λ_3}$$

$$\lambda' = \frac{\beta^e}{\beta}\mathcal{R}(l,\sigma)\lambda \tag{Λ_4}$$

where $\lambda' := u^{e\prime}(\bar{c}^e)/u_c(c,h)$, and $\mathcal{R}(l,\sigma) := \rho\ (= 1 + l\tau)$, where τ is a function of l, σ.

The competitive equilibrium labour supply can be expressed as follows, and is left out of the policymaker's problem:

$$h^{1-\alpha+\psi} c^{\gamma} = \alpha z(1-\tau) \tag{CE labour supply}$$

The policymaker's objective function can be expressed as follows:

$$V = \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\mu \beta^t \left(\frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\psi}}{1+\psi} \right) + \mu^e (\beta^e)^t \left(\log c_t^e + \frac{\log g(l_t, \sigma_t)}{1-\beta^e} \right) \right] \tag{B1}$$

The policymaker maximises (B3) subject to the constraints tagged by their respective Lagrange multipliers $(\Lambda_1, ..., \Lambda_4)$. The policymaker's programme can be convexified by log transformations when $\mathcal{R}, g$ are log-convex in l. Alternatively, constraint qualifications are straightforward to verify for more general specifications.

Envelope Condition

$$V_\lambda = \frac{\beta^e}{\beta} \mathcal{R}(l,\sigma) \Lambda_4$$

FONCs

$$\begin{aligned}
h: &\quad \mu h^{1-\alpha+\psi} = \alpha z \left[\Lambda_2 (1-\beta^e) \mathcal{R}(l,\sigma) - \Lambda_1 \right] \\
c: &\quad \mu = c^{\gamma} \left(\Lambda_3 \frac{\gamma}{c} c^{-\gamma} c^e - \Lambda_1 \right) \\
c^e: &\quad 0 = \mu^e + \Lambda_1 c^e - \Lambda_2 l c^e + \Lambda_3 c^{-\gamma} c^e \\
l: &\quad 0 = \frac{\mu^e}{1-\beta^e} \frac{g_l(l,\sigma)}{g(l,\sigma)} + \Lambda_2 \left[z h^{\alpha} (1-\beta^e) \mathcal{R}_l(l,\sigma) - c^e \right] + \Lambda_4 \frac{\beta^e}{\beta} \mathcal{R}_l(l,\sigma) \lambda \\
\lambda': &\quad 0 = -\Lambda_3 - \Lambda_4 + \beta \mathbb{E} \hat{V}_\lambda
\end{aligned}$$

where $\hat{V}$ is a placeholder.

By constraint 3

$$\begin{aligned} c: \qquad & \Lambda_1 = \Lambda_3 \frac{\gamma}{c} \lambda' - \mu c^{-\gamma} \\ c^e: \qquad & 0 = \mu^e + \Lambda_1 c^e - \Lambda_2 l c^e + \Lambda_3 \lambda' \end{aligned}$$

By constraint 2

$$c^e: \qquad \Lambda_2 (1-\beta^e) \mathcal{R}(l,\sigma) = \frac{1}{zh^\alpha} \left(\mu^e + \Lambda_1 c^e + \Lambda_3 \lambda'\right)$$

Combine FONCs for c, c^e:

$$\Lambda_2 (1-\beta^e) \mathcal{R} = \frac{1}{zh^\alpha} \left[\mu^e - \mu\lambda' + \Lambda_3 \lambda' \left(\gamma \frac{c^e}{c} + 1\right)\right]$$

B.1 Log utility

Assume Log utility, $\gamma = 1$:

$$h^{1-\alpha+\psi} c^\gamma = \alpha z \left[1 - \frac{\mathcal{R}(1-\beta^e)}{l} \left(1 - \frac{\mu^e}{\mu} \frac{1}{\lambda'}\right)\right]$$

$$h^{1-\alpha+\psi} c^\gamma = \alpha z (1-\tau) \left(1 + \frac{l\tau - \left(\frac{\lambda' - \lambda_0}{\lambda'}\right)(1-\beta^e)(1+l\tau)}{l(1-\tau)}\right)$$

where $\lambda_0 := \frac{\mu^e}{\mu}$.

The optimal wage subsidy ς^* is

$$\frac{l\tau - \left(\frac{\lambda' - \lambda_0}{\lambda'}\right)(1-\beta^e)(1+l\tau)}{l(1-\tau)}$$

B.2 Dynamics

Start from the optimal wage subsidy under log utility,

$$\varsigma^* = \frac{\tau}{1-\tau}\left(1 - \hat{\lambda}'(1-\beta^e)\frac{\rho}{\rho-1}\right)$$

where $\hat{\lambda}' := 1 - \frac{\lambda_0}{\lambda'}$. We can solve for $\hat{\lambda}'$,

$$\begin{aligned}
\frac{1}{\lambda'} &= \frac{c}{c^e} \\
&= \frac{y}{c^e} - 1 \\
&= \frac{l}{(1-\beta^e)\rho} - 1 \\
&= \frac{l - (1-\beta^e)\rho}{(1-\beta^e)\rho}
\end{aligned}$$

$$\begin{aligned}
\hat{\lambda}' &= 1 - \frac{(1-\beta^e)\rho_0}{l_0 - (1-\beta^e)\rho_0}\frac{l-(1-\beta^e)\rho}{(1-\beta^e)\rho} \\
&= 1 - \frac{(1-\beta^e)\rho_0(l-(1-\beta^e)\rho)}{(l_0-(1-\beta^e)\rho_0)(1-\beta^e)\rho} \\
&= \frac{(l_0-(1-\beta^e)\rho_0)(1-\beta^e)\rho - (1-\beta^e)\rho_0(l-(1-\beta^e)\rho)}{(l_0-(1-\beta^e)\rho_0)(1-\beta^e)\rho} \\
&= \frac{l_0 - l\frac{\rho_0}{\rho}}{(l_0-(1-\beta^e)\rho_0)} \\
&= \left(1 - \frac{l\rho_0}{l_0\rho}\right)\frac{l_0}{(l_0-(1-\beta^e)\rho_0)}
\end{aligned}$$

Substituting into the expression for optimal wage subsidies yields

$$\varsigma^* = \frac{\tau}{1-\tau}\left(1 - \left(\frac{l_0\rho - l\rho_0}{\rho-1}\right)\frac{(1-\beta^e)}{l_0-(1-\beta^e)\rho_0}\right) \tag{B2}$$

B.3 Optimal policy under discretion (Proof of Proposition 2)

Under discretion, the following equilibrium and market clearing conditions are constraints faced by the policymaker. First, the aggregate resource constraint:

$$zh^{\alpha} \geq c + c^{e} \tag{Λ_1}$$

Second, the policymaker takes entrepreneurial net wealth q^{e} brought into the period as given. Risk free real interest rates r are fixed by assumption, ultimately implying that policymakers take the opportunity cost of net wealth, n^{e}, as given. Leverage is therefore increasing in aggregate production:

$$ln^{e} = zh^{\alpha} \tag{Λ_2}$$

where $\mathcal{R}(l,\sigma) := \rho\ (= 1 + l\tau)$, where τ is a function of l, σ.
The policymaker's objective function can be expressed as follows:

$$V = \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\mu\beta^{t} \left(\frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\psi}}{1+\psi} \right) + \mu^{e}(\beta^{e})^{t} \left(\log c_t^{e} + \frac{\log g(l_t,\sigma_t)}{1-\beta^{e}} \right) \right] \tag{B3}$$

The first order conditions are

$$\begin{aligned}
h: \quad & \frac{\mu h^{\psi}}{\alpha z h^{\alpha-1}} = \Lambda_1 - \Lambda_2 \\
c: \quad & \frac{\mu}{c^{\gamma}} = \Lambda_1 \\
c^{e}: \quad & \frac{\mu^{e}}{c^{e}} = \Lambda_1 \\
l: \quad & 0 = \Lambda_2 n^{e} + \frac{\mu^{e}}{1-\beta^{e}} \frac{g_l(l_t,\sigma_t)}{g(l_t,\sigma_t)}
\end{aligned}$$

$$\frac{c^{\gamma}h^{\psi}}{\alpha z h^{\alpha-1}} = 1 + \frac{c^{\gamma}\mu^{e}}{\mu} \left(\frac{1}{1-\beta^{e}} \frac{g_l(l_t,\sigma_t)}{n^{e} g(l_t,\sigma_t)} \right)$$

$$\frac{c^{\gamma}h^{\psi}}{\alpha z h^{\alpha-1}} = 1 + \frac{g_l(l_t,\sigma_t)}{g(l_t,\sigma_t)} \mathcal{R}(l,\sigma)$$

$$\frac{c^\gamma h^\psi}{\alpha z h^{\alpha-1}} = 1 - \mathcal{T}(l, \sigma)$$

Which is the same as in the competitive allocation in the absence of policy. Given that the policymaker does not introduce a wage subsidy within the period, there is no effect of policy on intertemporal trade, leaving allocations unchanged relative to the competitive equilibrium.